The Association
of
Engineering and Shipbuilding Draughtsmen.

GEAR MANUFACTURE.

BOOK 4—STRAIGHT BEVEL GEARS.

Straight Bevel, Design.
Straight Bevel, Manufacture.
Internal Spur, Design.
Internal Spur, Shaping and Slotting.

By F. WHITELEY.

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Also published—

"GEAR MANUFACTURE."

By F. WHITELEY.

BOOK 1.

Spur gears—internal spur gears—spur rack—helical hears—double helical gears—spiral gears—bevel gears—spiral bevel gears—worm gears—notations—involite form—construction of involute form—generation of tooth form—involute function—correction of spur gears—stub tooth—increasing pressure angle—positive and negative correction — extended centre distance gearing — involute tooth action—specific sliding. (30 diagrams).

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BOOK 2.

Spur gears — detail design — addendum and dedendum—pitch—pressure angle — backlash — vernier settings — constant chord settings — form milling — dividing head — differential indexing—hobbing — shaping — rack planing — profile grinding—formers—generation. (22 diagrams).

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BOOK 3.

Spiral and helical gears—detail design—normal pitch—transverse pitch—axial pitch—tooth proportions—vernier settings—virtual number of teeth—backlash—pressure angle—spiral gears, fixed centres—thrust diagrams—form milling—cutter selection—layout of tooth shape—indexing—hobbing—hand of hob—rack planing—gear shaping—cutter diameter—tooth proportions—profile grinding—pitch block diameter.

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BOOK 4.

STRAIGHT BEVEL GEARS.

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GEAR MANUFACTURE.

By F. Whiteley.

CHAPTER I.

STRAIGHT BEVEL GEARS.

Detail Design.

The tooth characteristics of a pair of straight bevel gears on the transverse plane, generally as shown in Fig. 7, Book I, are very similar to a pair of spur gears.

Instead of the teeth however being the same section across the whole width of face, the tooth profiles and bottom land all meet at one point commonly termed the apex point.

As previously explained, the tooth form on the transverse plane is for all practical purposes an involute, and the gear teeth can be set out or calculations made relative to the form in exactly the same manner as for spur gears.

Therefore, if a layout of the form at the front end of the teeth was required, it would be exactly the same as that on the transverse plane but on a reduced scale, the proportion being in the ratio of the inside cone distance divided by the outside cone distance C measured on the pitch cone line.

When inside cone distance $= C - F$

and C = outside cone distance or distance from transverse plane to apex point.

F = width of face.

With this method it is thus possible to obtain the tooth shape at any desired position between the transverse plane and apex point, the proportions being obtained by dividing the distance from the apex to the plane on which the shape is required, by the cone distance C measured on the pitch cone line.

This feature of bevel gear design is very important, for if the tooth dimensions are to be produced to the sizes given on the drawing, it is imperative that the back angle face is machined to the correct cone distance, and to maintain this distance the outside diameter J and tip distance must both be held to a maximum tolerance of 0·002 as shown in Fig. 1.

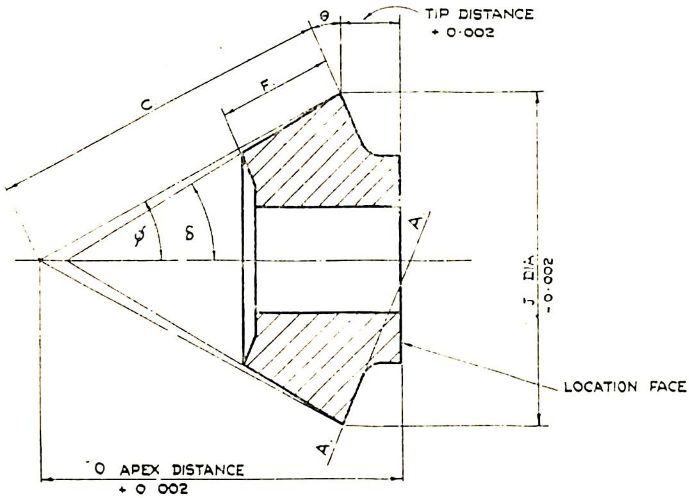


FIG I SHEWING DIMENSIONS AND TOLERANCES FOR OBTAINING ACCURATE CONE DISTANCE

When θ = back cone angle
ϕ = gauge angle
δ = face angle

For gears which have to be ground on the end location face after hardening it is again advisable to make the tip distance to a tolerance of 0·002, as by so doing the gears can be assembled much more easily, and if suitable gauges are provided the tolerance given can be maintained without much difficulty.

In any case should the gears exceed the tolerance given the actual apex distance should be etched on each gear to enable them to be meshed together in their correct running position with the aid of suitable shims.

Pitch.

For bevel gears the term DP or diametral pitch is now almost universally used, the definition of this term in words being the number of teeth per inch of pitch circle diameter, or expressed as a formula

$$\mathrm{DP} = \frac{Z}{D}$$

When DP = diametral pitch
Z = actual number of teeth
D = pitch circle diameter

or given Z and DP

$$D = \frac{Z}{DP} \text{ inches}$$

The circular pitch or module pitch could be used as in the case of spur gears, but one standard will be found satisfactory for all requirements although it would be an advantage to specify the module on the gear drawing in addition to the addendum, as it will then be possible to determine whether the gear tooth is of standard proportions, by the following rules.

(*a*) If addendum of gear is the same as the module then the tooth form is of standard proportions.

(*b*) If the addendum is greater than the module the gear tooth has positive correction, or if the addendum is less than the module a negative correction, the amount of correction being the difference between the two values.

$$\text{When module} = \frac{1}{DP}$$

Pressure Angle.

The British Standard for the pressure angle of bevel gears is 20°, but the Gleason Co. on whose machines the majority of these gears are cut, recommend a basic pressure angle of $14\frac{1}{2}°$, which has certain advantages over higher pressure angles namely :—

(*a*) Larger contact ratio.
(*b*) Less effect of eccentricity.
(*c*) Smaller radial and axial thrust loads.

Unfortunately, however, these advantages can only be obtained for gears with sufficiently large numbers of teeth, as gears with small numbers of teeth of standard proportions are produced with undercut flanks, which reduce tooth contact, and strength.

To eliminate the possibility of undercut, pressure angles of $17\frac{1}{2}°$ and 20° are also used, while for ratios over 1·000 the addendum of the wheel is made smaller than the module and the addendum of the pinion larger by the same amount. This is due to the fact that ratios having a long addendum pinion and a short addendum wheel have more action in recess than in approach (with pinion driving), have stronger pinion teeth, and the lower pressure angle can be used without the pinion flanks being undercut.

Long and short addendum correction however cannot be used for a pair of bevel gears having equal numbers of teeth, *i.e.*, having a ratio of 1·000, and in this case the addendum and dedendum are made standard, and undercut eliminated by increasing the pressure angle.

The minimum number of teeth which can be cut without undercut, given the pressure angle, for a ratio of 1·000 can be obtained from the formula

$$Z_v = \frac{1{\cdot}000 \times 2}{\sin^2 \psi}$$

When Z_v = virtual number of teeth.
ψ = pressure angle on transverse plane.
and Z_v = $Z \sec \theta$.
When Z = actual number of teeth.
θ = pitch angle.

Now as the pitch angle will always, of course, be equal to 45° for a bevel gear pair having equal numbers of teeth, the virtual number of teeth Z_v will be equal to

$$Z \sec 45^\circ$$

and having obtained the value Z_v from the formula previously given the actual number of teeth Z can be obtained as follows :—

$$Z = Z_v \cos 45^\circ$$

Thus the minimum actual number of teeth which can be cut without undercut, given the pressure angle, for a pair of gears having equal numbers of teeth can be obtained from the formula

$$Z = \frac{(1{\cdot}000 \times 2)}{\sin^2 \psi} \cos 45^\circ$$

For 14½°, 17½° and 20° pressure angle gears the minimum numbers of teeth will be 23, 16 and 12 teeth respectively.

Pitch Angle.

The pitch angle of any bevel gear is the angle between the axis and the cone generating line, denoted by θ_p or θ_w, and given the numbers of teeth in the wheel and pinion gearing at a shaft angle of 90°,

$$\tan \theta_w = \frac{Z_1}{Z_2} \text{ and } \theta_p = 90^\circ - \theta_w$$

When θ_w = pitch angle of wheel.
θ_p = pitch angle of pinion.
Z_1 = number of teeth in wheel.
Z_2 = number of teeth in pinion.

Tooth Proportions.

For all ratios other than 1·000 the addendum of the pinion is increased and the wheel addendum decreased by the same amount, this usually being termed positive and negative correction.

Instead of making the circular tooth thickness proportionately greater for the pinion, and less for the wheel, the method recommended by the Gleason Co. provides wheel and pinion teeth of approximately equal strength, although the sum of the two values obtained must equal the pitch in inches or

$$(t_w + t_p) \quad = \quad \frac{\pi}{\text{DP}} \quad \text{which is termed the circular pitch.}$$

when t_w = circular tooth thickness of wheel.
t_p = circular tooth thickness of pinion.

To obtain the working depth, whole depth, pressure angle, wheel addendum, and tooth thickness of the wheel, various tables are used, particulars of which are given in Figs. 2, 3 and 4, these tables only being used for straight bevel gears having generated teeth, gearing at right angles, and having 13 or more pinion teeth.

For bevel gears mounted at shaft angles other than 90° the pinion, *i.e.*, the gear having the smaller number of teeth, should have primary consideration with regard to the amount of correction required to eliminate undercut, and reference should also be made to the specification of the machine to determine whether or not the gear can be cut, the limitations generally being decided upon by the pitch angle of the pinion.

14½° PRESSURE ANGLE	17½° PRESSURE ANGLE	20° PRESSURE ANGLE
RATIOS WITH	RATIOS	RATIOS
27 OR MORE TEETH IN PINION	26/26 TO 26/29	18/18 TO 8/20
26/30 AND HIGHER	25/25 TO 25/34	17/17 TO 17/25
25/35 AND HIGHER	24/24 TO 24/47	16/16 TO 16/100
24/48 AND HIGHER	23/23 TO 23/100	15/15 TO 15/100
	22/22 TO 22/100	14/16 TO 14/100
	21/21 TO 21/100	13/23 TO 13/100
	20/20 TO 20/100	
	19/19 TO 19/100	
	18/21 TO 18/100	
	17/26 TO 17/100	

FIG. 2. TABLE OF PRESSURE ANGLES TO BE USED WHEN GIVEN RATIO OF BEVEL GEAR PAIR.

FIG. 3. WHEEL ADDENDUMS FOR 1 DIAMETRAL PITCH

RATIOS		ADD^M (A)	RATIOS		ADD^M (A)	RATIOS		ADD^M (A)	RATIOS		ADD^M (A)
FROM	TO		FROM	TO		FROM	TO		FROM	TO	
1·00	1·00	1·000	1·15	1·17	0·880	1·42	1·45	0·760	2·06	2·16	0·640
1·00	1·02	0·990	1·17	1·19	0·870	1·45	1·48	0·750	2·16	2·27	0·630
1·02	1·03	0·980	1·19	1·21	0·860	1·48	1·52	0·740	2·27	2·41	0·620
1·03	1·04	0·970	1·21	1·23	0·850	1·52	1·56	0·730	2·41	2·58	0·610
1·04	1·05	0·960	1·23	1·25	0·840	1·56	1·60	0·720	2·58	2·78	0·600
1·05	1·06	0·950	1·25	1·27	0·830	1·60	1·65	0·710	2·78	3·05	0·590
1·06	1·08	0·940	1·27	1·29	0·820	1·65	1·70	0·700	3·05	3·41	0·580
1·08	1·09	0·930	1·29	1·31	0·810	1·70	1·75	0·690	3·41	3·94	0·570
1·09	1·11	0·920	1·31	1·33	0·800	1·76	1·82	0·680	3·94	4·82	0·560
1·11	1·12	0·910	1·33	1·36	0·790	1·82	1·89	0·670	4·82	6·81	0·550
1·12	1·14	0·900	1·36	1·39	0·780	1·89	1·97	0·660	6·81	∞	0·540
1·14	1·15	0·890	1·39	1·42	0·770	1·97	2·06	0·650	—	—	—

IN CASE OF CHOICE USE THE LARGER ADDENDUM.

NUMBER OF TEETH IN PINION	RATIOS														
	1·00 TO 1·25	1·25 TO 1·50	1·50 TO 1·75	1·75 TO 2·00	2·00 TO 2·25	2·25 TO 2·50	2·50 TO 2·75	2·75 TO 3·00	3·00 TO 3·25	3·25 TO 3·50	3·50 TO 3·75	3·75 TO 4·00	4·00 TO 4·50	4·50 TO 5·00	5·00 AND HIGHER
	VALUES OF CONSTANT 'K' IN INCHES FOR DIFFERENT RATIOS														
13	0·000	0·015	0·040	0·045	0·050	0·060	0·070	0·080	0·090	0·100	0·110	0·120	0·135	0·150	0·165
14	0·000	0·015	0·030	0·050	0·065	0·080	0·090	0·100	0·110	0·120	0·125	0·130	0·140	0·150	0·160
15 TO 17	0·000	0·000	0·010	0·020	0·030	0·045	0·060	0·070	0·080	0·090	0·095	0·100	0·110	0·115	0·120
18 TO 21	0·000	0·000	0·000	0·000	0·010	0·030	0·045	0·060	0·070	0·080	0·085	0·090	0·095	0·100	0·100
22 TO 29	0·000	0·000	0·000	0·000	0·010	0·030	0·040	0·050	0·060	0·065	0·070	0·070	0·080	0·085	0·085
30 & UP	0·000	0·000	0·000	0·000	0·010	0·025	0·035	0·040	0·045	0·050	0·055	0·060	0·065	0·070	0·070

FIG 4. TABLE OF CONSTANT 'K'
FOR DIFFERENT RATIOS

To use the tables it is first of all necessary to calculate the working depth, whole depth and the ratio, then let

h_k = working depth of tooth.
h_t = whole depth of tooth.
R = ratio.

and $h_k = \dfrac{2{\cdot}000}{DP}$ for all pitches and

$h_t = \dfrac{2{\cdot}188}{DP}$ for large and medium pitches up to 20 DP.

and $h_t = \dfrac{2{\cdot}250}{DP}$ for pitches 20 DP and finer.

while $R = \dfrac{\text{number of teeth in wheel}}{\text{number of teeth in pinion}}$ or $\dfrac{Z_1}{Z_2}$

After having calculated the ratio value the pressure angle ψ is obtained from the table, Fig. 2, and the wheel addendum A from the value given in table, Fig. 3, divided by the diametral pitch, and in case of choice the larger addendum should be used.

The pinion addendum a is then obtained from the formula

$$a = h_k - A$$

and $b = h_t - a$ and $B = h_t - A$
when a = addendum of pinion.
b = dedendum of pinion.
A = addendum of wheel.
B = dedendum of wheel.

Tables are also used to obtain the constant K which is used in the formula to calculate the circular tooth thickness of the wheel the formula for each pressure angle being given below.

$$t_w = \frac{1{\cdot}071}{DP} + (0{\cdot}5 \times A) - \frac{K}{DP} \quad \text{when } \psi = 14\tfrac{1}{2}^\circ$$

$$t_w = \frac{0{\cdot}971}{DP} + (0{\cdot}6 \times A) - \frac{K}{DP} \quad \text{when } \psi = 17\tfrac{1}{2}^\circ$$

$$t_w = \frac{0{,}871}{DP} + (0{\cdot}7 \times A) - \frac{K}{DP} \quad \text{when } \psi = 20^\circ$$

The constant K is obtained from table, Fig. 4, taking into consideration the ratio and the number of teeth in the pinion, and in case of doubt the smaller value for K should be used.

After having obtained the value t_w from one of the above formulae the circular tooth thickness of the pinion is equal to

$$t_p = \frac{\pi}{DP} - t_w$$

As previously stated the British Standard for bevel gears recommends a standard pressure angle of 20°, and this has an advantage over the Gleason system if it is desired to keep stocks of cutters as low as possible. Correction is used to avoid undercutting of the pinion, when the virtual number of teeth Z_v in the pinion is 19 or less, the addendum for both pinion and wheel being given below.

Virtual number of teeth in pinion.	Corrected Addendum 1 DP 20° Pressure Angle.	
	Pinion (a)	Wheel (A)
20 or above	1·00	1·00
19	1·04	0·96
18	1·08	0·92
17	1·12	0·88
16	1·16	0·84
15	1·20	0·80
14	1·24	0·76
13	1·28	0·72
12	1·32	0·68
11	1·36	0·64
10	1·40	0·60

The tooth thickness of both pinion and wheel are then made greater and less respectively according to the amount of correction, then

$$t_p = \frac{\pi}{DP} + 2\,(K \tan 20^\circ)$$

$$t_w = \frac{\pi}{DP} - 2\,(K \tan 20^\circ)$$

when t_p = circular tooth thickness of pinion.

t_w = circular tooth thickness of wheel.

K = correction factor = a − module.

The whole depth of tooth h_t being made the same as that used in the Gleason system.

It should be specially noted, however, that the amount of correction is based on the virtual number of teeth Z_v and not the actual number of teeth Z as used in the Gleason system, while if the sum of the virtual numbers of teeth of a pair of bevel gears is less than 40 the tooth design should have special consideration as positive correction of the pinion may introduce undercutting of the wheel flanks.

Cone Distance.

The cone distance C is the distance from the apex to pitch circle diameter D on the transverse plane and by calculation

$$C = \frac{D}{2 \sin \theta_w} \quad \text{or} \quad C = \frac{d}{2 \sin \theta_p}$$

when D = pitch circle diameter of wheel.
d = pitch circle diameter of pinion.
θ_w = pitch angle of wheel.
θ_p = pitch angle of pinion.
C = cone distance.

Addendum Angle.

The addendum angle α is obtained from the cone distance and addendum on the transverse plane from the formula

$$\tan \alpha = \frac{A}{C}$$

When A = addendum on transverse plane.
C = cone distance.

Dedendum Angle.

Similarly the dedendum angle β is obtained from the cone distance and dedendum on the transverse plane from the formula

$$\tan \beta = \frac{B}{C}$$

when B = dedendum on transverse plane.
C = cone distance.

Root Angle.

This angle is very important as it is the angle to which the generating machine is set when cutting the gear teeth.

Then let θ = pitch angle.
β = dedendum angle.
γ = root angle.
and $\gamma = \theta - \beta$.

Gauge Angle.

The angle denoted by ϕ in Fig. 1 is equal to the sum of the pitch angle and the addendum angle or expressed as a formula

$\phi = \theta + \propto$

when ϕ = gauge angle.
θ = pitch angle.
$\propto$ = addendum angle.

This angle, usually termed the face angle in the majority of text-books, is the angle between the axis of the gear and the tip of the tooth.

If it is desired to turn the blanks to the gauge angle it will be found that when gears are in mesh the clearance in the roots of the teeth (although equal to $\dfrac{0{\cdot}188}{DP}$ or $\dfrac{0{\cdot}250}{DP}$ on the transverse plane AA) will gradually decrease towards the apex of the teeth.

Face Angle.

In order to maintain uniform clearance in the roots of the teeth the gear blanks can be turned on the face to a face angle equal to δ, as shown in Fig. 1.

It should be specially noted, however, that this angle bears no relationship to the apex point as is the case with all other angles and is obtained from the value of the dedendum angle of the mating gear.

When θ_w = pitch angle of wheel.
θ_p = pitch angle of pinion.
β_p = dedendum angle of pinion.
β_w = dedendum angle of wheel.
δ_w = face angle of wheel.
δ_p = face angle of pinion.

Then $\delta_w = \theta_w + \beta_p$ or $\delta_p = \theta_p + \beta_w$

To illustrate this point a pair of bevel gears is shown in Fig. 5 and it will be noted that the face cone angle is turned parallel to the root cone of the mating gear, in order to eliminate possible fillet interference at the small ends of the teeth.

Tooth Angle.

When using the generating method of producing the teeth of bevel gears a cutter having an inclined face equal to the pressure angle of the gear to be cut is generally used.

The work head is set at an angle equal to the root angle γ, to the cutter head, and the cutter slides are then set to the tooth angle calculated on the rack tooth thickness at the root of the tooth, in relation to the cone distance as shown in Fig. 6.

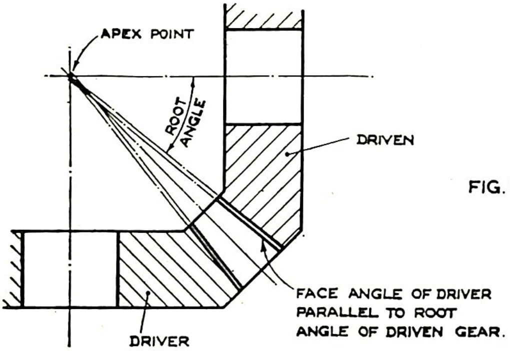


FIG. 5. SHEWING RELATIVE POSITION OF ROOT AND FACE ANGLES OF TWO GEARS IN MESH.

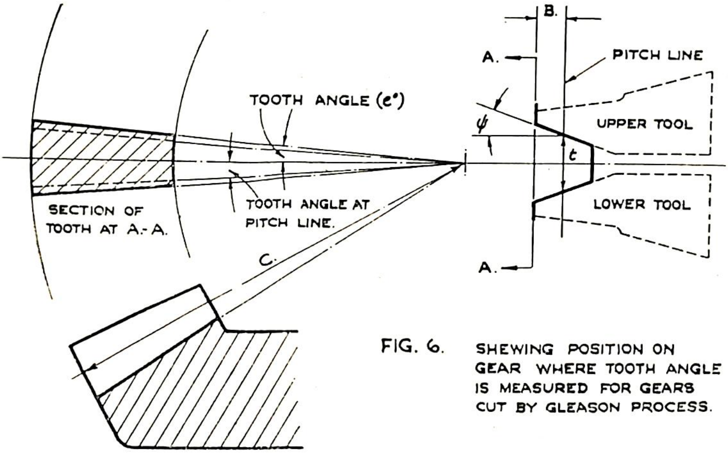


FIG. 6. SHEWING POSITION ON GEAR WHERE TOOTH ANGLE IS MEASURED FOR GEARS CUT BY GLEASON PROCESS.

The circular tooth thickness "t" has also to be taken into consideration and wheels and pinions having different numbers of teeth will have different values for the tooth angle setting.

Let t = circular thickness of tooth on pitch circle diameter.
C = cone distance.
B = dedendum of gear.
ψ = pressure angle.
e = tooth angle.

Then

$$\tan e = \frac{t/2 + (\mathrm{B} \tan \psi)}{\mathrm{C}}$$

Cutter Point Width.

The cutters used are made a definite width of flat on the front or points, this width being such that the side of the cutter opposite the pressure angle face does not touch the opposite side of the tooth space.

The width of flat must therefore be limited according to the pitch or the tooth thickness of the gear to be cut and should not be greater than the calculated width obtained from the formula given below.

$$P_p = \frac{C - F}{C} (t_w - 2b \tan \psi) \text{ or}$$

$$P_w = \frac{C - F}{C} (t_p - 2B \tan \psi)$$

when P_p = width of tooth space in root at front end of face of pinion.
P_w = width of tooth space in root at front end of face of wheel.
t_w = circular thickness of wheel tooth.
t_p = circular thickness of pinion tooth.
b = dedendum of pinion.
B = dedendum of wheel.
ψ = pressure angle of cutter.

Face Width.

In order to keep the cutter point width as large as possible the face width of the gears should be made not more than one third of the cone distance. In fact in good design the ratio of face width to the cone distance should be from 0·25 to 0·3.

Increasing the face width over recommended proportions may add strength and durability theoretically, but at a rapidly diminishing rate, as on wide face gears there is a danger of wear if the load becomes concentrated on the small ends of the teeth.

Therefore given C = cone distance.
$F = C \times 0{\cdot}3$ or $0{\cdot}25$.

Outside Diameter.

To obtain the outside diameter of a bevel gear the pitch diameter and addendum must be known then

$$J = D + 2 (\cos \theta_w A).$$

or $j = d + 2 (\cos \theta_p a)$.

When J = outside diameter of wheel.
j = outside diameter of pinion.
D = pitch circle diameter of wheel.
d = pitch circle diameter of pinion.
θ_w = pitch angle of wheel.
θ_p = pitch angle of pinion.
A = addendum of wheel.
a = addendum of pinion.

Tip Distance.

The tip distance is the distance from the tip of the tooth to the back locating face whether as mounted for cutting or running.

To obtain this distance it is first necessary to obtain the distance termed pitch line to tip "L" or "*l*" and this is obtained from the formula

$$L = A \sin \theta_w \qquad \text{or } l = a \sin \theta_p$$

to which must be added

$$O_w - C \cos \theta_w \qquad \text{or } O_p - C \cos \theta_p$$

to obtain the tip distance of the wheel or pinion, when

O_w = Apex distance of wheel or distance from back location face to apex.
O_p = Apex distance of pinion or distance from back location face to apex.
θ_w = pitch angle of wheel.
θ_p = pitch angle of pinion.
C = cone distance.
L = distance pitch line to tip wheel.
l = distance pitch line to tip pinion.

Back Cone Distance.

This distance is used when it is desired to make a layout of the tooth shape, make calculations in connection with the tooth form, or calculate the pitch line vernier settings.

It is generally termed the back cone radius and is made equal to $\dfrac{d_v}{2}$ or $\dfrac{D_v}{2}$ when $\dfrac{d_v}{2} = C \tan \theta_p$ or $\dfrac{D_v}{2} = C \tan \theta_w$

when D_v = virtual pitch circle diameter on back angle face of wheel.
d_v = virtual pitch circle diameter on back angle face of pinion.
C = cone distance.
θ_p = pitch angle of pinion.
θ_w = pitch angle of wheel.

Virtual Number of Teeth.

The virtual number of teeth is the number of teeth the same pitch as the gear which corresponds to the virtual pitch circle diameter D_v or d_v, and is obtained from the formula

$$Z_{v1} = Z_1 \sec \theta_w \qquad \text{or } Z_{v2} = Z_2 \sec \theta_p$$

when Z_v = virtual number of teeth.
Z = actual number of teeth.
θ_w = pitch angle of wheel.
θ_p = pitch angle of pinion.

Vernier Settings.

The vernier settings of a bevel gear pair of equal ratio can be obtained from the table given in Chapter I, Book 2, for spur gears, by substituting the number of teeth in the table by the virtual number of teeth, and dividing the values given by the diametral pitch.

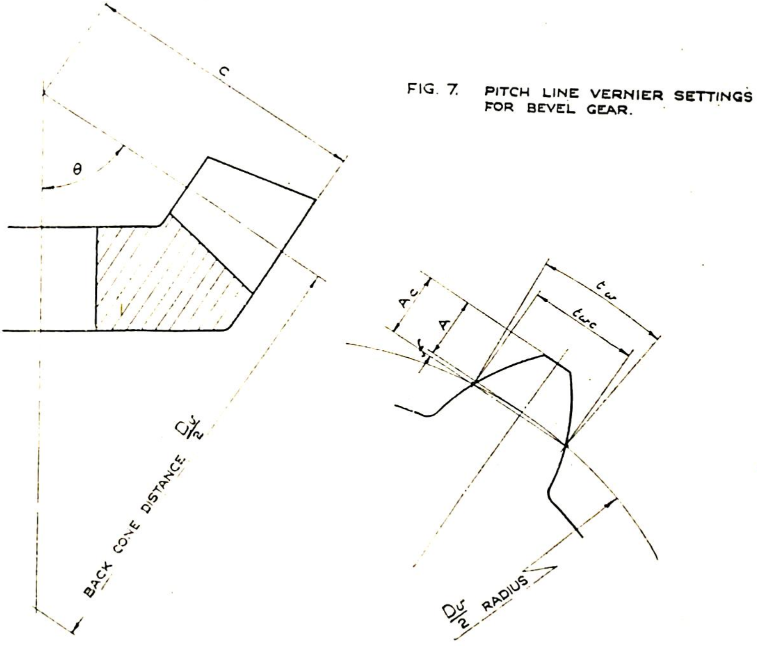


FIG. 7. PITCH LINE VERNIER SETTINGS FOR BEVEL GEAR.

For other ratios, however (the teeth having been corrected), it will first be necessary to know the addendum and circular tooth thickness of the gear for which the vernier settings are required, then referring to Fig. 7 let

A = addendum of wheel.
t_w = circular tooth thickness of wheel tooth on diameter D.
D_v = virtual pitch circle diameter of wheel.
$\propto$ = half-angle subtended by t_w.
f = correction factor.
A_c = chordal addendum of wheel tooth.
t_{wc} = chordal thickness of wheel tooth.

Then $\dfrac{t_w}{D_v} = \propto$ radians.

convert $\propto$ radians to degrees when one radian $= 57{\cdot}296°$

$$\text{and } t_{wc} = \sin \propto D_v.$$

$$\text{then} = \frac{D_v}{2} \text{ versine } \propto.$$

$$\text{and } A_c = A + f.$$

A quicker method for calculating the chordal addendum and thickness for bevel gears than that described above is to use the formulae given below, as recommended by the Gleason Co. :—

$$t_{wc} = t_w - \frac{t^3{}_w}{6\,D^2}$$

$$t_{pc} = t_p - \frac{t^3{}_p}{6d^2}$$

$$A_c = A + \frac{t^2{}_w \cos \theta_w}{4D}$$

$$a_c = a + \frac{t^2{}_p \cos \theta_p}{4d}$$

the same notation being used as for the first method when

D = actual pitch circle diameter of wheel.
d = actual pitch circle diameter of pinion.

Backlash.

From the value t_{wc} must be deducted half the backlash allowance, which can be the same as that given for spur gears in Chapter I, Book 2.

A drawing should be made for both wheel and pinion similar to that shown in Fig. 8, the actual cutting particulars (with the exception of the root angle which can be shown on the drawing) being neatly tabulated on the right hand side of the drawing.

Example.

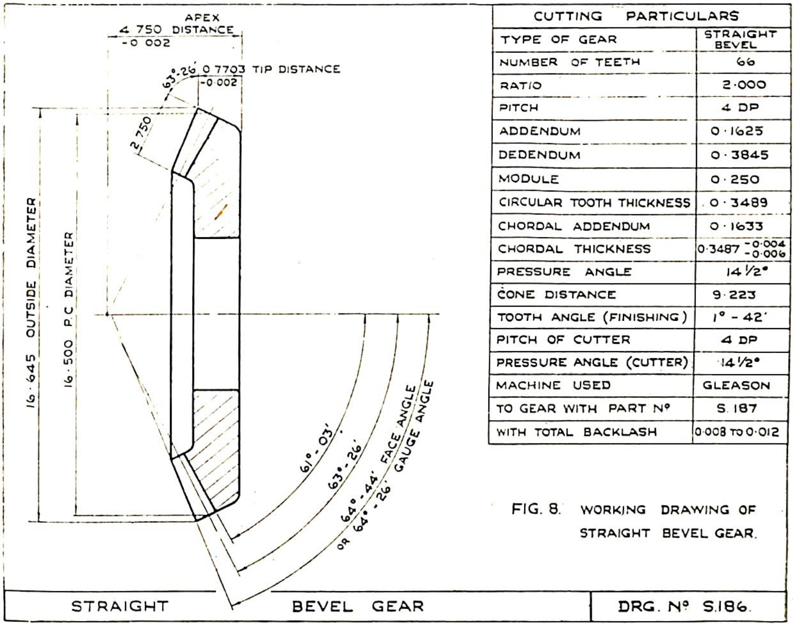
An example for the detailed design of a pair of straight bevel gears is given below, the particulars given being as follows :—

$$Z_1 = 66 \qquad Z_2 = 33 \qquad DP = 4$$

Pitch Angle.

$$\tan \theta_w = \frac{66}{33} = 2{\cdot}000 \qquad \theta_p = 90° - 63° - 26'$$

$$= 63° - 26' \qquad\qquad = 26° - 34'$$



CUTTING PARTICULARS	
TYPE OF GEAR	STRAIGHT BEVEL
NUMBER OF TEETH	66
RATIO	2·000
PITCH	4 DP
ADDENDUM	0·1625
DEDENDUM	0·3845
MODULE	0·250
CIRCULAR TOOTH THICKNESS	0·3489
CHORDAL ADDENDUM	0·1633
CHORDAL THICKNESS	$0{\cdot}3487\,^{-0{\cdot}004}_{-0{\cdot}006}$
PRESSURE ANGLE	14½°
CONE DISTANCE	9·223
TOOTH ANGLE (FINISHING)	1° - 42′
PITCH OF CUTTER	4 DP
PRESSURE ANGLE (CUTTER)	14½°
MACHINE USED	GLEASON
TO GEAR WITH PART Nº	S. 187
WITH TOTAL BACKLASH	0·008 TO 0·012

FIG. 8. WORKING DRAWING OF STRAIGHT BEVEL GEAR.

Pitch Circle Diameter.

$$D = \frac{66}{4} = 16{\cdot}500 \qquad d = \frac{33}{4} = 8{\cdot}250$$

Ratio.

$$R = \frac{Z_1}{Z_2} = \frac{66}{33} = 2{\cdot}000$$

Working Depth.

$$h_k = \frac{2{\cdot}000}{DP} = \frac{2{\cdot}000}{4} = 0{\cdot}500$$

Whole Depth.

$$h_t = \frac{2{\cdot}188}{DP} = \frac{2{\cdot}188}{4} = 0{\cdot}547$$

Pressure Angle.

From table, Fig. 2, make $\psi = 14\frac{1}{2}^\circ$ when R = 2·000

Gear Addendum.

From table, Fig. 3, using ratio value of 2·000

$$A = \frac{0{\cdot}650}{4} = 0{\cdot}1625$$

Pinion Addendum.

$$a = \frac{2{\cdot}000}{DP} - A = \frac{2{\cdot}000}{4} - 0{\cdot}1625 = 0{\cdot}3375$$

Dedendum.

$$B = \frac{2{\cdot}188}{DP} - A = \frac{2{\cdot}188}{4} - 0{\cdot}1625 = 0{\cdot}3845$$

$$\text{or } b = \frac{2{\cdot}188}{DP} - a = \frac{2{\cdot}188}{4} - 0{\cdot}3375 = 0{\cdot}2095$$

Tooth Thickness.

When $\psi = 14\tfrac{1}{2}°$

$t_w = \frac{1{\cdot}071}{DP} + (0{\cdot}5 \times A) - \frac{K}{DP}$ when value K is obtained from table, Fig. 4.

$$= \frac{1{\cdot}071}{4} + (0{\cdot}5 \times 0{\cdot}1625) - 0$$

$$= 0{\cdot}2677 + 0{\cdot}0812$$

$$= 0{\cdot}3489 \text{ and } t_p = \frac{\pi}{DP} - 0{\cdot}3489$$

$$= 0{\cdot}7854 - 0{\cdot}3489$$

$$= 0{\cdot}4365$$

Cone Distance.

$$C = \frac{16{\cdot}500}{2 \sin 63''\!-\!26'} = 9{\cdot}223$$

$$\text{or } C = \frac{8{\cdot}250}{2 \sin 26'' - 34'} = 9{\cdot}223$$

Addendum Angle.

$$\tan \propto_w = \frac{A}{C} \qquad \text{or } \tan \propto_p = \frac{a}{C}$$

$$\begin{aligned} &= \frac{0{\cdot}1625}{9{\cdot}223} \qquad\qquad && = \frac{0{\cdot}3375}{9{\cdot}223} \\ &= 0{\cdot}01761 && = 0{\cdot}03659 \\ &= 1^\circ\ 00' && = 2^\circ\ 06' \end{aligned}$$

Dedendum Angle.

$$\begin{aligned} \tan\beta_w &= \frac{B}{C} \qquad\qquad & \text{or } \tan\beta_p &= \frac{b}{C} \\ &= \frac{0{\cdot}3845}{9{\cdot}223} & &= \frac{0{\cdot}2095}{9{\cdot}223} \\ &= 0{\cdot}04168 & &= 0{\cdot}02271 \\ &= 2^\circ\ 23' & &= 1^\circ\ 18' \end{aligned}$$

Root Angle.

$$\begin{aligned} \gamma_w &= 63^\circ\ 26' - 2^\circ\ 23' \qquad\qquad & \gamma_p &= 26^\circ\ 34' - 1^\circ\ 18' \\ &= 61^\circ\ 03' & &= 25^\circ\ 16' \end{aligned}$$

Gauge Angle.

$$\begin{aligned} \phi_w &= \theta_w + \propto_w \qquad\qquad & \phi_p &= \theta_p + \propto_p \\ &= 63^\circ\ 26' + 1^\circ\ 00' & &= 26^\circ\ 34' + 2^\circ\ 06' \\ &= 64^\circ\ 26' & &= 28^\circ\ 40' \end{aligned}$$

Face Angle.

$$\begin{aligned} \delta_w &= \theta_w + \beta_p \qquad\qquad & \delta_p &= \theta_p + \beta_w \\ &= 63^\circ\ 26' + 1^\circ\ 18' & &= 26^\circ\ 34' + 2^\circ\ 23' \\ &= 64^\circ\ 44' & &= 28^\circ\ 57' \end{aligned}$$

Tooth Angle.

$$\tan e_w = \frac{\frac{t_w}{2} + (B \tan\theta)}{C} \qquad\qquad \tan e_p = \frac{\frac{t_p}{2} + (b \tan\theta)}{C}$$

$$= \frac{\frac{0{\cdot}4365}{2} + (0{\cdot}2095 \times 0{\cdot}25862)}{9{\cdot}223}$$

$$= \frac{\frac{0{\cdot}3489}{2} + (0{\cdot}3845 \times 0{\cdot}25862)}{9{\cdot}223}$$

$$\begin{aligned} &= 0{\cdot}02969 \qquad && = 0{\cdot}02953 \\ &= 10^\circ\ 42'\ 4'' && = 1^\circ\ 41'\ 30'' \end{aligned}$$

Face Width.

$$\begin{aligned} F &= C \times 0{\cdot}3 \\ &= 9{\cdot}223 \times 0{\cdot}3 \\ &= 2{\cdot}7669 \text{ make } 2{\cdot}750 \end{aligned}$$

Cutter Point Width.

$$\begin{aligned} P_w &= \frac{C - F}{C}\,(t_p - 2B \tan \psi) \\ &= \frac{9{\cdot}223 - 2{\cdot}767}{9{\cdot}223}\,(0{\cdot}4365 - 0{\cdot}769 \times 0{\cdot}25862) \\ &= 0{\cdot}700 \times 0{\cdot}2377 \\ &= 0{\cdot}1663 \end{aligned}$$

$$\begin{aligned} P_p &= \frac{C - F}{C}\,(t_w - 2b \tan \psi) \\ &= \frac{9{\cdot}223 - 2{\cdot}767}{9{\cdot}222}\,(0{\cdot}3489 - 0{\cdot}419 \times 0{\cdot}25862) \\ &= 0{\cdot}700 \times 0{\cdot}2407 \\ &= 0{\cdot}1685 \end{aligned}$$

from which a standard 4 DP $14\frac{1}{2}°$ pressure angle cutter can be used as the point width of the cutter is 0·130.

Outside Diameter.

$$\begin{aligned} J &= D + 2\,(\cos \theta_w\, A) \\ &= 16{\cdot}500 + 2\,(0{\cdot}44724 \times 0{\cdot}1625) \\ &= 16{\cdot}645 \end{aligned}$$

$$\begin{aligned} j &= d + 2\,(\cos \theta_p\, a) \\ &= 8{\cdot}250 + 2\,(0{\cdot}89441 \times 0{\cdot}3375) \\ &= 8{\cdot}853 \end{aligned}$$

Tip Distance.

$$\begin{aligned} L &= A \sin \theta_w \\ &= 0{\cdot}1625 \times 0{\cdot}89441 \\ &= 0{\cdot}1453 \end{aligned}$$

$$\begin{aligned} l &= a \sin \theta_p \\ &= 0{\cdot}3375 \times 0{\cdot}44724 \\ &= 0{\cdot}1509 \end{aligned}$$

Then tip distance :—

wheel $= L + (O_w - C \cos \theta_w)$ pinion $= l + (O_p - C \cos \theta_p)$

when O_w and O_p is the horizontal distance from the back location face to the apex then let $O_w = 4{\cdot}750$ and $O_p = 8{\cdot}875$

then tip distance (wheel)

$$\begin{aligned} &= 0{\cdot}1453 + (4{\cdot}750 - 4{\cdot}125) \\ &= 0{\cdot}7703 \end{aligned}$$

and tip distance (pinion)

$$\begin{aligned} &= 0{\cdot}1509 + (8{\cdot}875 - 8{\cdot}250) \\ &= 0{\cdot}7759 \end{aligned}$$

Back Cone Radius.

$$\begin{aligned} \frac{D_v}{2} &= C \tan \theta_w \\ &= 18{\cdot}446 \end{aligned}$$

$$\begin{aligned} \frac{d_v}{2} &= C \tan \theta_p \\ &= 4{\cdot}6115 \end{aligned}$$

Virtual Number of Teeth.

$$\begin{aligned} Z_{v1} &= Z_1 \sec \theta_w \\ &= 66 \times 2{\cdot}2359 \\ &= 147{\cdot}5 \end{aligned} \qquad \begin{aligned} Z_{v2} &= Z_2 \sec \theta_p \\ &= 33 \times 1{\cdot}1180 \\ &= 36{\cdot}89 \end{aligned}$$

Vernier Settings.

$$\begin{aligned} \frac{t_w}{D_v} &= \frac{0{\cdot}3489}{36{\cdot}892} \\ &= 0{\cdot}00945733 \text{ radians} \\ \alpha_w &= 0^\circ\ 32'\ 30'' \\ t_{wc} &= \sin \alpha_w\, D_v \\ &= 0{\cdot}0094192 \times 36{\cdot}892 \\ &= 0{\cdot}3487 \\ f_w &= \frac{D_v}{2} \text{ versine } \alpha_w \\ &= 18{\cdot}446 \times 0{\cdot}0000447 \\ &= 0{\cdot}00081 \\ A_c &= A + f_w \\ &= 0{\cdot}1625 + 0{\cdot}00081 \\ &= 0{\cdot}1633 \end{aligned} \qquad \begin{aligned} \frac{t_p}{d_v} &= \frac{0{\cdot}4365}{9{\cdot}223} \\ &= 0{\cdot}0474791 \text{ radians} \\ \alpha_p &= 2^\circ\ 42'\ 42'' \\ t_{pc} &= \sin \alpha_p\, d_v \\ &= 0{\cdot}0474599 \times 9{\cdot}223 \\ &= 0{\cdot}4363 \\ f_p &= \frac{d_v}{2} \text{ versine } \alpha_p \\ &= 4{\cdot}6115 \times 0{\cdot}0011269 \\ &= 0{\cdot}00516 \\ a_c &= a + f_p \\ &= 0{\cdot}3375 + 0{\cdot}00516 \\ &= 0{\cdot}3427 \end{aligned}$$

Using the method recommended by the Gleason Co. we have

$$\begin{aligned} t_{wc} &= 0{\cdot}3489 - \frac{0{\cdot}3489^3}{6 \times 16{\cdot}5^2} \\ &= 0{\cdot}3489 - 000026 \\ &= 0{\cdot}3489 \\ t_{pc} &= 0{\cdot}4365 - \frac{0{\cdot}4365^3}{6 \times 8{\cdot}25^2} \\ &= 0{\cdot}4365 - 0{\cdot}0002 \\ &= 0{\cdot}4363 \\ A_c &= 0{\cdot}1625 + \frac{0{\cdot}3489^2 \times \cos 63^\circ\ 26'}{4 \times 16{\cdot}5} \\ &= 0{\cdot}1625 + 0{\cdot}000825 \\ &= 0{\cdot}16332 \\ a_c &= 0{\cdot}3375 + \frac{0{\cdot}4365^2 \times \cos 26^\circ\ 34'}{4 \times 8{\cdot}25} \\ &= 0{\cdot}3375 + 0{\cdot}0051641 \\ &= 0{\cdot}34266 \end{aligned}$$

which gives the same result as obtained by the previous method.

Backlash from Table.

0·008 to 0·012.

CHAPTER II.

STRAIGHT BEVEL GEARS.

Manufacture.

To obtain really first class cut bevel gears, the first essential is to provide gear blanks having the face angle, back angle and outside diameter machined to fairly close limits, while the gear blank must also be concentric to the hole or shank used for mounting the gear when the teeth are being cut.

This, of course, applies to all other types of gears, but for bevel gears inaccurately machined blanks cause endless trouble on inspection and assembly.

As previously explained in Chapter I the size of tooth specified is only correct on the back angle face or transverse plane (unless otherwise stated) when the position of this face has been previously determined from the pitch and number of teeth in the gear.

Therefore, if, for example, blanks are produced with this face out of position, or having any variation on the outside diameters, the size of tooth produced will not conform to the tooth sizes given on the drawings, and in Fig. 9, different examples are shown of

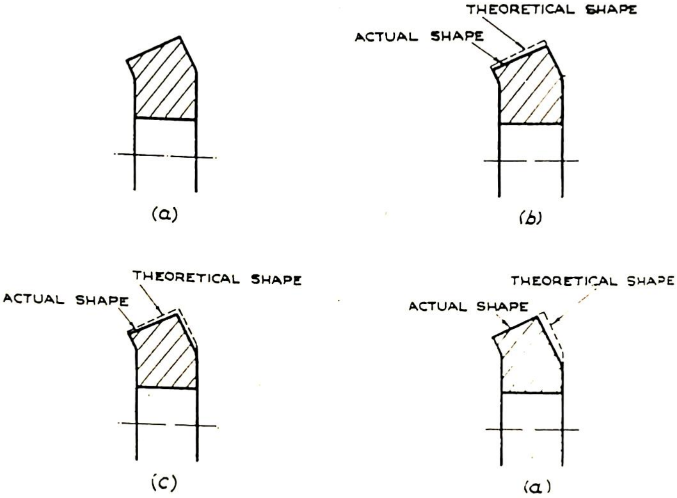


FIG 9 SHEWING EXAMPLES OF BEVEL GEAR BLANKS INCORRECTLY TURNED.

incorrectly turned blanks, and the effect each one has on the size of tooth produced.

At (*a*) a sketch of a correctly turned blank is first shown, while at (*b*) the face angle although machined to the correct angle is out of position, thus reducing the outside diameter and increasing the tip distance, therefore in this instance the teeth would be thick to vernier settings, and when meshed there would be excessive root clearance.

At (*c*) both the face angle and back angle are below nominal and this would have the effect of obtaining excessive root clearance and bad meshing, due to the back angle face being out of position. In the case of the gear blank shown at (*d*) the teeth would be found to be thick when measured with the gear tooth verniers, while the gear would not be meshed correctly, and it will, therefore, be apparent that bevel gears should be meshed only at correct apex distances, thus ensuring that the pitch cones of the two gears meet at the common apex point.

Gear blanks should therefore be machined as near as possible to the theoretical dimensions and a tolerance of 0·002 should be worked to for the outside diameter and tip distance dimensions.

The tolerance of 0·002 on the two dimensions in question is easily obtainable if the gear blank is mounted in a suitable fixture on a centre lathe or capstan lathe in the same manner as is used for cutting the teeth.

Taking for example a gear which is located on the shank, the angles are first rough turned leaving an allowance of 0·020 to 0·025 on the face and back angles.

The shank is ground on centres to a location limit, then, locating this shank and holding back the gear blank by means of a suitable drawbar the two angles are skimmed to the finished sizes given on the drawing, a special fixture being used similar to that shown diagrammatically in Fig. 10.

The slide A is set to equal the back angle of the gear and slide B to the face angle, by means of gauges or a protractor, the gauges locating against pins C and D or C and E, pin C being removable to enable the slides to operate after setting.

The angle turning fixture is mounted on the cross slide of the machine, and to obtain the correct blank outside diameter J the fixture is set by means of pin C in relation to the centre line of the machine to a distance equal to J/2, and the correct tip distance is obtained by setting pin C the correct distance from the locating face of the work holding fixture using slip gauges, and it will therefore be noted that when the fixture is set in the correct position the centre of pin C corresponds to the tip of the gear blank.

The clock indicator is then set at zero at this position with the saddle locked, and to remove the work the saddle is unlocked and

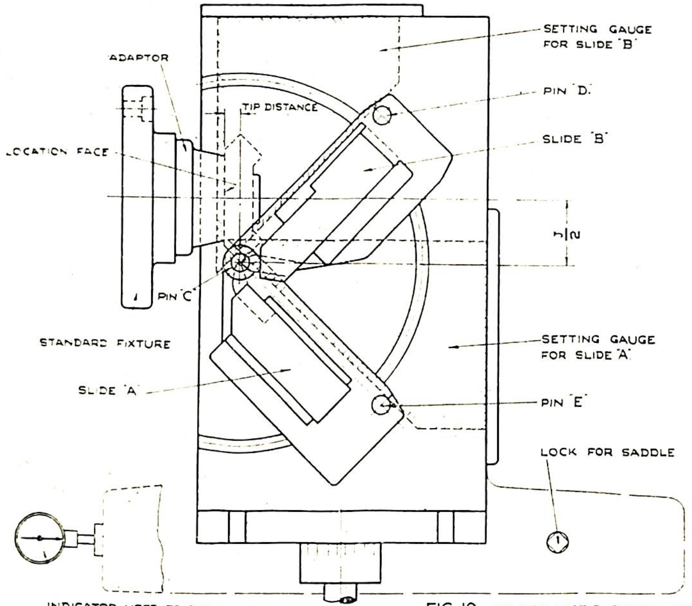


FIG. 10. DIAGRAMMATIC SKETCH OF SPECIAL FIXTURE FOR TURNING FACE AND BACK ANGLES OF BEVEL GEAR BLANK BEFORE TOOTH CUTTING

moved to the right, another blank mounted, the saddle brought back to zero, locked, and the two angles finish turned.

Providing that everything has been set accurately the blanks will conform in every way to the tolerance and concentricity required, and when cut the teeth will all be the same size and cut at the same setting, thus ensuring interchangeability.

The Machine.

Various makes of machines are in existence for cutting straight bevel gear teeth, but the most popular and by far superior is the Gleason bevel gear generator.

For small lot production it is common practice to first rough cut the teeth at one setting, after which the machine is then changed over to the finishing operation.

For mass production, however, a special Gleason roughing machine is now available, named the Revex machine, on which large diameter milling cutters are used, the cutter form closely corresponding to the finished form of tooth space to be cut. With this machine production is very high and only a small amount of stock is left on the tooth flanks for finishing, thus the gear teeth can be finished much more quickly.

Alternatively the teeth can be rough cut on a horizontal milling machine using a dividing head or a special fixture similar to the one shown in Fig. 11, but in either case the angle at which the head is set must be equal to the root angle.

To obtain the correct depth of tooth a setting gauge corresponding to the root cone of the gear is usually supplied, and mounted in the same way as the gear to be cut.

The roughing depth should wherever possible be made slightly deeper than the finishing depth, so that the finishing tools have only to cut the tooth flanks, and when using this method about 0·010 to 0·015 should be left on each flank for finishing, the same amount being allowed when the teeth are rough cut on the Gleason generator.

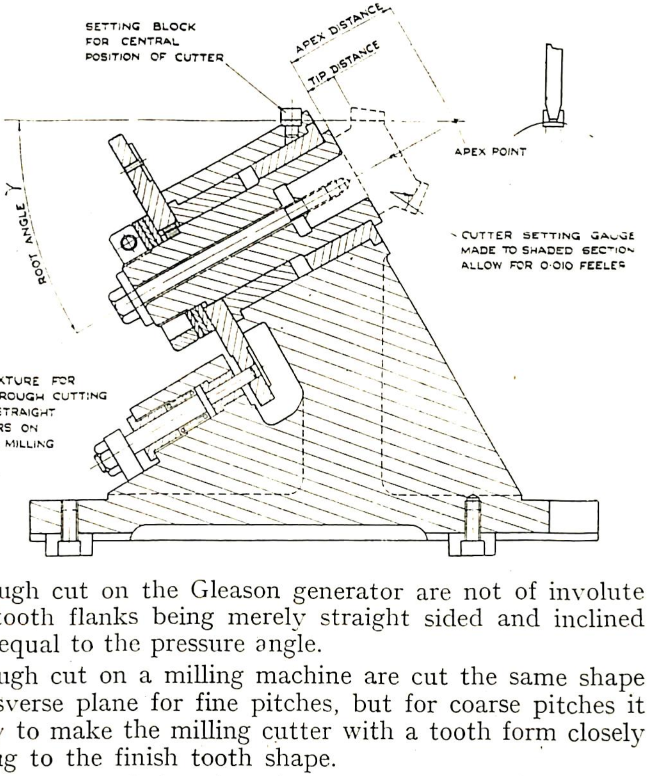

SETTING BLOCK FOR CENTRAL POSITION OF CUTTER
APEX DISTANCE
TIP DISTANCE
APEX POINT
ROOT ANGLE Y
CUTTER SETTING GAUGE MADE TO SHADED SECTION ALLOW FOR 0·010 FEELER

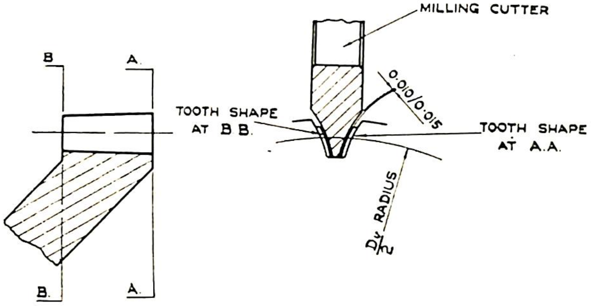
FIG. 11 SPECIAL FIXTURE FOR USE WHEN ROUGH CUTTING TEETH OF STRAIGHT BEVEL GEARS ON HORIZONTAL MILLING MACHINE

Gears rough cut on the Gleason generator are not of involute shape, the tooth flanks being merely straight sided and inclined at an angle equal to the pressure angle.

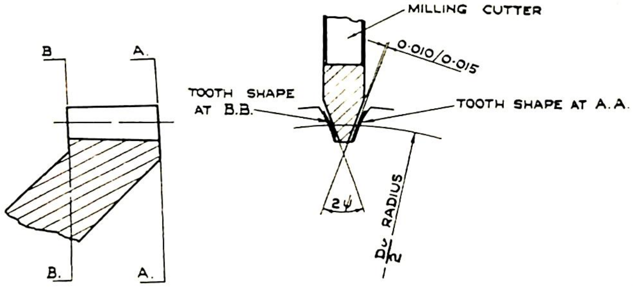
Gears rough cut on a milling machine are cut the same shape on the transverse plane for fine pitches, but for coarse pitches it is customary to make the milling cutter with a tooth form closely corresponding to the finish tooth shape.

When the shape and size of tooth space at the front end differs greatly from the tooth space on the transverse plane it is not economical to rough cut the teeth on a milling machine ; in this case the teeth should either be rough cut on a Revex roughing machine or Gleason generator.

When deciding upon the shape of the milling cutter (which should be cam relieved) the front end of the tooth space should be taken into consideration and a layout of the tooth space at both front and rear ends of the tooth made, similar to that shown in Fig. 12.



FORMED CUTTER SHAPE FOR COARSE PITCHES



FORMED CUTTER SHAPE FOR FINE PITCHES

FIG. 12. SKETCH OF MILLING CUTTERS USED FOR ROUGH CUTTING STRAIGHT BEVEL GEARS.

A compromise is made between the two shapes to decide upon the shape of the milling cutter which will be suitable for cutting the tooth space as near the finished size as possible, an allowance being made on the thickness of the cutter to leave stock on the tooth flanks for the finishing operation.

In other words the gear blanks should be rough cut to enable the gear teeth to be finish cut in one cut, to avoid having to adjust the tooth angle slides on the generating machine, and this should be possible for all bevel gears up to about 4 DP and when the face width is not more than one third the cone distance.

Instructions for setting the finishing machine are given in the handbooks supplied with each machine, but great care must be exercised when designing the work holding equipment.

The most common error is to make the adaptor or arbor used to mount the gear blank in the spindle, of insufficient length from the spindle face of the machine, and this is especially so in the case of small pinions.

It is therefore again advisable to make a layout of the machine similar to the one shown in Fig. 13, the layout giving the maximum and minimum distances from the spindle face, according to the pitch angle of the gear, also showing the clearance between the cutter slides and spindle when the work head is set on the minimum pitch angle.

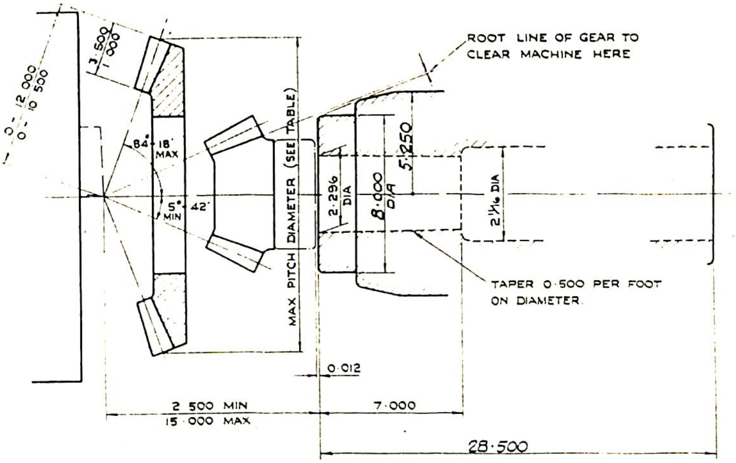


FIG. 13. TOOLING LAYOUT FOR 12" STRAIGHT BEVEL GEAR GENERATOR

It will be apparent from the layout that the smaller the pitch angle the greater must be the distance from the spindle face to the apex point, but even so the distance must be kept to a minimum to hold the gear blank as rigid as possible during the cutting operation.

The arbors or adaptors should be checked for concentricity while in position in the machine, and the locating face should also be checked for squareness, before the gears are cut.

Another important point is the head setting on the machine, this being made equal to the sum of the apex distance of the gear to be cut, and the distance from the location face of the arbor to the spindle face of the machine or referring to Fig. 14.

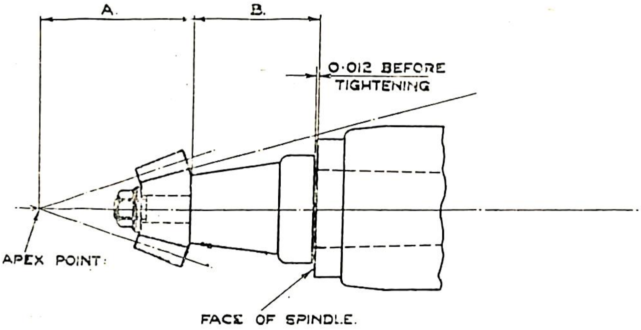


FIG. 14. SKETCH SHEWING DIMENSIONS REQUIRED FOR HEAD SETTING OF GLEASON GENERATOR.

Head Setting = A + B

dimension B being measured when the arbor is drawn tight into the spindle, while dimension A is obtained from the drawing of the gear blank at the stage before cutting the teeth.

Instructions for setting the machine for both roughing and finishing operations are given in the instruction book supplied with each machine, care being taken to make each setting as accurate as possible.

Double Index Roughing.

Double indexing or indexing two teeth per index can be used for the roughing operation only and reduces the cutting time for this operation by about 40%.

Special cutters are required, however, and slight adjustments have to be made to the tooth angle, and root angle, while the index change gears have to be arranged to index half the number of teeth to be cut.

This is very simple when dealing with a gear having an even number of teeth, but when double index change gears are required for a gear having an odd number of teeth the gear ratio for the actual number of teeth is first written down and one of the drivers is changed to a gear having twice the number of teeth or one of the driven gears is changed to a gear having half the number of teeth as shown in the examples below :—

Example—93 Teeth to be Cut.

Index Gears from chart for 93 teeth single indexing. $\frac{36}{62} \times \frac{50}{90} = \frac{\text{drivers}}{\text{driven}}$

Index Gears used when double indexing 93 teeth. $\frac{72}{62} \times \frac{50}{90} = \frac{\text{drivers}}{\text{driven}}$

Example—25 Teeth to be Cut.

Index Gears from chart for 25 teeth single indexing.

$$\frac{64}{56} \times \frac{63}{60} = \frac{\text{drivers}}{\text{driven}}$$

Index Gears used when double indexing 25 teeth.

$$\frac{80}{35} \times \frac{63}{60} = \frac{\text{drivers}}{\text{driven}}$$

The next step is to calculate the tooth angle setting, root angle, and the pressure angles of the cutters, and referring to Fig. 15 we have :—

D = correction angle.
A = outside angle of cutter.
B = inside angle of cutter.
C = root angle of gear.
E = machine root angle.
F = 180° divided by the number of teeth to be cut.
G = tooth angle setting.
P = point width of cutters.
H = pressure angle of gear.

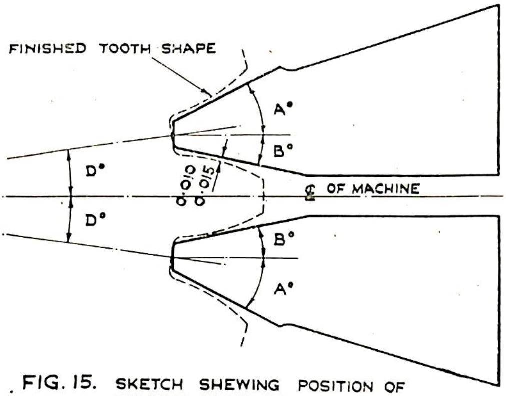


FIG. 15. SKETCH SHEWING POSITION OF DOUBLE INDEX ROUGHING TOOLS IN RELATION TO FINISHED TOOTH SHAPE.

Then—

$$\sin D = \sin F \cos C.$$
$$A = H + D.$$
$$B = H - D.$$

and

$$\tan E = \tan C \cos F.$$
$$\sin G = \sin D \tan C.$$

The point width P of the cutters should then be calculated as previously explained in Chapter I, based on the tooth thickness for roughing and when setting the cutters to the angle gauge used for the finishing cutters they should be set closer to the centre by an amount equal to half the point width. A slight adjustment may, however, be necessary, to obtain the same tooth thickness of two adjacent teeth. The double indexing method can be used for all bevel gears having 30 teeth and above, but it is not recommended for gears having less than 30 teeth.

Special Pressure Angles.

Although three standard pressure angles, namely, $14\frac{1}{2}^\circ$, $17\frac{1}{2}^\circ$ and 20° have been adopted for bevel gear design it is still possible to cut other pressure angles using standard cutters, for the finishing operations.

It is preferable, however, to use a cutter having the same pressure angle as the gear to be cut, but if this is not possible a cutter can be used to cut a pressure angle having a slightly larger pressure angle than the cutter being used.

For example, should it be necessary to cut a gear $17\frac{1}{2}^\circ$ pressure angle, and no cutters are available, then a cutter having a pressure angle of $14\frac{1}{2}^\circ$ can be used, or if a gear pressure angle of $22\frac{1}{2}^\circ$ is required a 20° pressure angle cutter could be used. As explained in Book I, the pressure angle of an involute form is variable and can only be determined when the pitch circle diameter is known, therefore if a bevel gear is say $22\frac{1}{2}^\circ$ on the pitch circle diameter it is possible to determine the position on the tooth flank where the pressure angle of the same tooth is 20°.

For bevel gears this new position can be determined in terms of the pitch angle and when

θ = pitch angle of gear.
ψ = pressure angle of gear.
ψ_c = pressure angle of cutter.
θ_N = pitch angle where gear is equal to cutter pressure angle.
A = difference between θ and θ_N

$$\tan A = \left(1 - \frac{\cos \psi}{\cos \psi_c}\right) \tan \theta$$

$$\text{and } \theta_N = \theta^\circ - A^\circ$$

This value θ_N is then used in the formula to obtain another set of ratio roll change gears using the compound change gear ratio tables, or in other words the roll change gear ratio is based on the pitch angle where the gear is the same pressure angle as the cutter being used.

Then let R = decimal ratio of roll change gears.

θ_N = pitch angle where gear is equal to cutter pressure angle.

Z = number of teeth in gear.

$$\text{and } R = \frac{\text{drivers}}{\text{driven}} = \frac{Z}{75 \sin \theta_N}$$

Cutters.

Finishing and roughing cutters used for single indexing are practically identical excepting that the finishing cutters have side relief, and roughing cutters have not.

The cross section of the cutters is the same and after calculating the point width of the cutter as explained in Chapter I, the design of the cutter is fairly straightforward.

In Fig. 16 is shown a sketch of a pair of cutters and it will be noted that the pitch of the cutter depends on the point width and width of cutting edge, the overall height of the cutters being the same in all cases.

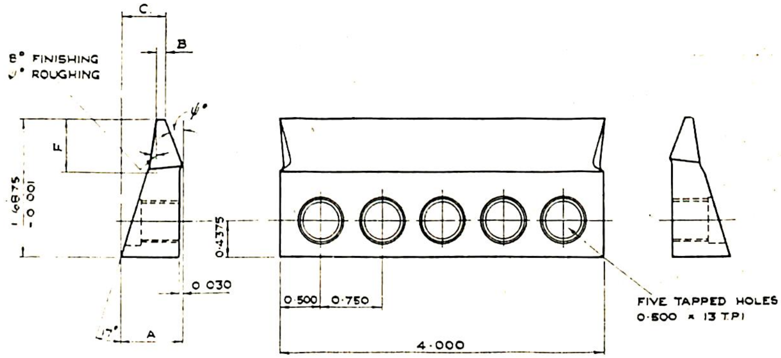


FIG 16 SKETCH OF CUTTERS FOR 12" GLEASON STRAIGHT BEVEL GEAR GENERATOR

The value "C" is also a constant dependent upon the pressure angle of the cutter, but it is not very important due to the fact that the cutter holder on the machine is adjustable.

Given whole depth of tooth h_t

$$F = h_t + \frac{0{\cdot}200}{DP} \text{ when}$$

DP = diametral pitch.

A = $C + F \tan \psi$.

ψ = pressure angle.

B = point width of cutter.

the dimension B being made slightly less than the limit or cutter point width calculated in Chapter I.

Bedding.

After finish cutting the teeth it is advisable to mount the gear with a master gear or mating gear on a testing machine, and run the gears under light load to obtain the marking or bedding.

The two gears should be set at the correct apex distances with the specified amount of backlash and if the bedding is not satisfactory in this position it is possible to detect where the defect lies from the bedding obtained.

Full instructions for making the necessary adjustments, covering different examples of bedding are given in the instruction book supplied with the machine.

CHAPTER III.

INTERNAL SPUR GEARS.

Detail Design.

A most common error made in connection with the design of internal spur gears is to accept the rather misleading statement made in many text books, that all dimensions for internal gears may be found by using the same formulae as those for external spur gears.

Exceptions are made in the case of the internal diameter, which takes the place of the outside diameter for a spur gear, and the centre distance, which for internal gearing is equal to the difference between the two pitch circle radii, and not the sum.

Internal spur gears have, it is agreed, been designed in this way many times in the past, but the results obtained are far from satisfactory, while the difficulties met with in producing the gear teeth on a production basis make it an almost impossible proposition.

The main difficulty is met with when producing the internal gear teeth by the gear shaper method, as in this case the size of the cutter used imposes certain restrictions on the tooth proportions of the internal gear.

If the cutter used is too large the tips of two or more teeth in the internal gear will be trimmed as the cutter is fed into depth, and to avoid this small special cutters are used, yet on the other hand if the cutter is too small imperfect tooth forms will be produced on the internal gear.

Another factor to be considered in the design of internal gears, in order to secure effective tooth action, is the question of interference.

There are three possible ways in which interference can take place on an internal gear drive, namely :—

Involute interference.
Tip interference.
Fillet interference.

and these must be avoided if a satisfactory drive is to be obtained.

With regard to involute interference this is present when all the working profile of the internal tooth is not of involute form, but by increasing the internal diameter and displacing the tooth form outwards away from the centre of the internal gear, the required condition can be obtained.

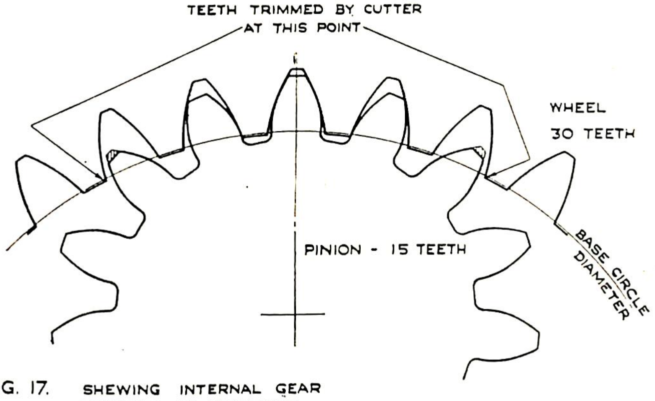


FIG. 17. SHEWING INTERNAL GEAR DESIGNED TO STANDARD PROPORTIONS.

Tip interference will exist when the tips of the spur pinion interfere with the tips of the internal gear as the teeth come into and go out of mesh, and to eliminate the trouble the spur pinion must be made sufficiently smaller than the internal gear.

Finally fillet interference exists when the tips of one member interfere with the fillets or radii in the roots of the teeth in the other member, and proper tooth proportions must be selected to avoid this interference.

It is, therefore, considered that more effort is required when designing internal gears than is the case for external spur gears, not overlooking the fact that standard cutters should be used in all cases.

In Fig. 17 is shown an internal gear pair of 15/30 ratio designed in accordance with the standard formula for external spur gears when their proportions for both wheel and pinion are as follows :—

Addendum $\frac{1}{DP}$

Dedendum $\frac{1 \cdot 157}{DP}$

Circular Tooth Thickness $\frac{1 \cdot 5708}{DP}$

Pressure Angle 20°

It will be noted that all the defects previously mentioned are present, and that it would be impossible to produce the internal gear teeth by the generating process.

As in the case of the external spur gear the drawings of both the internal wheel and pinion should give all the relevant data, to enable the gears to be produced without having to refer to any other source for information, the data being neatly tabulated as shown in Figs. 18 and 19.

Pitch.

As in the case of spur gears the pitch used is the diametral pitch or DP, indicating the number of teeth per inch of pitch diameter.

Pitch Diameter.

The pitch diameters of both wheel and pinion are calculated as follows :—

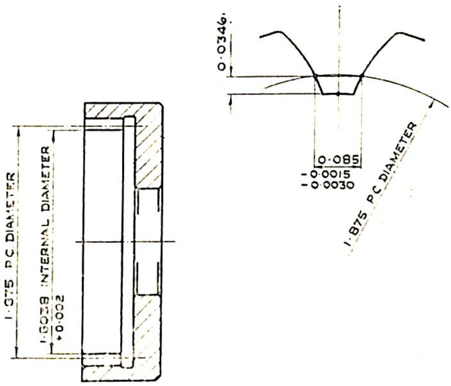
$$D = \frac{Z_1}{DP} \qquad \text{and } d = \frac{Z_2}{DP}$$

when D = pitch diameter of internal gear.
d = pitch diameter of pinion.
Z_1 = number of teeth in internal gear.
Z_2 = number of teeth in pinion.

Gear Centres.

The gear centres must be reduced in order to increase the running clearance or backlash in the case of internal gears, therefore the tolerances on centre distance should be in accordance with the table below.

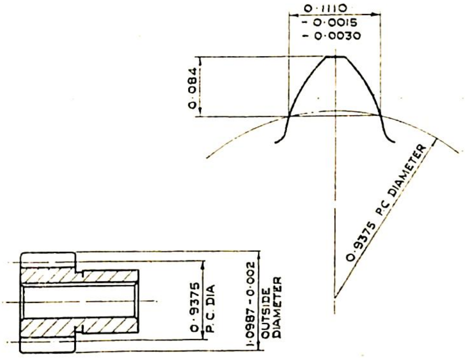
Diametral Pitch	*Centre Distance Tolerance*
16 and finer	0·000 to — 0·002
14 to 5	0·000 to — 0·003
4 to 2	0·000 to — 0·004
1¾ to 1	0·000 to — 0·005



CUTTING PARTICULARS	
TYPE OF GEAR	INTERNAL SPUR
NUMBER OF TEETH	30
ADDENDUM	0·0356
DEDENDUM	0·0962
PITCH	16 DP
MODULE	0·0625
PRESSURE ANGLE	20°
CIRCULAR THICKNESS	0·0850
CHORDAL ADDENDUM	0·0346
CHORDAL THICKNESS	0·085 $^{-0·0015}_{-0·0030}$
RUNNING CENTRES	0·4687 - 0·002
WITH MATING GEAR	B 626
BACKLASH	0·003/0·006
CUTTER N°	
PITCH OF CUTTER	16 DP
PRESSURE ANGLE	20°

FIG. 18. DETAIL DRAWING OF INTERNAL SPUR GEAR

INTERNAL SPUR GEAR



CUTTING PARTICULARS	
TYPE OF GEAR	SPUR
NUMBER OF TEETH	15
ADDENDUM	0·0806
DEDENDUM	0·060
PITCH	16 DP
MODULE	0·0625
PRESSURE ANGLE	20°
CIRCULAR THICKNESS	0·1113
CHORDAL ADDENDUM	0·084
CHORDAL THICKNESS	0·1110 $^{-0·0015}_{-0·0030}$
RUNNING CENTRES	0·4687 - 0·002
WITH MATING GEAR	B 627
BACKLASH	0·003/0·006
CUTTER N°	STANDARD
PITCH OF CUTTER	16 DP
PRESSURE ANGLE	20°

FIG 19 DETAIL DRAWING OF EXTERNAL SPUR PINION.

SPUR PINION

and the centre distance C will equal

$$\frac{D - d}{2} \qquad \text{or} \qquad \frac{Z_1 - Z_2}{2\ DP}$$

when D = pitch diameter of internal gear.
d = pitch diameter of pinion.
Z_1 = number of teeth in internal gear.
Z_2 = number of teeth in pinion.
DP = pitch of gear.

Addendum and Dedendum and Pressure Angle.

No standard rule can be given for the addendum and dedendum of either the internal gear or pinion if the gears are designed correctly from both the point of view of the designer and production.

Certain rules have to be made which are entirely dependent on the number of teeth in the pinion, but in the case of the pressure angle this can be definitely fixed at 20° for all internal gear drives as this angle appears to be the most effective in overcoming interference.

First taking the spur pinion into consideration the addendum must be made greater than 1/DP, and the tooth thickness increased in proportion, the pinion tooth being given what is termed positive correction in accordance with the value given in Table 1 when addendum will equal.

$$\frac{\text{Pinion addendum from Table 1}}{DP}$$

Knowing the amount the standard addendum has been increased the enlarged circular tooth thickness of the pinion can be calculated as follows :—

Let K = amount pinion addendum is increased.
ψ = pressure angle.
t_p = circular tooth thickness of pinion tooth.

$$\text{Then } t_p = \frac{3{\cdot}1416}{2\ DP} + 2\left(\frac{K}{DP}\tan\psi\right)$$

Values for K are given in the table below for 1 DP, and for other pitches the value given must be divided by the diametral pitch.

TABLE 1.

No. of teeth in pinion	10	11	12	13	14	15	16 and above
Correction Constant K	·500	·460	·415	·375	·440	·290	·250
Pinion Addendum	1·500	1·460	1·415	1·375	1·330	1·290	1·250

To allow the enlarged pinion tooth to mesh with the internal gear the circular tooth thickness of the internal gear on the pitch line is reduced accordingly then when t_w = circular tooth thickness of internal gear on pitch line.

$$t_w = \frac{3{\cdot}1416}{2\ \mathrm{DP}} - 2\left(\frac{K}{\mathrm{DP}}\tan\psi\right) \quad \text{or} \quad t_w = \frac{3{\cdot}1416}{\mathrm{DP}} - t_p$$

The dedendum of the pinion tooth is made less than standard by an amount equal to K/DP to enable a cutter of standard proportions to be used, thus the whole depth of tooth will be standard, and as a gear shaper cutter should be used for cutting the teeth the whole depth of tooth of the pinion will be equal to

$$\frac{2{\cdot}250}{\mathrm{DP}}$$

The wheel dedendum will be equal to the pinion addendum plus clearance or expressed as a formula when B = wheel dedendum and b = pinion dedendum.

$$B = \frac{1{\cdot}250}{\mathrm{DP}} + \frac{K}{\mathrm{DP}} \qquad b = \frac{1{\cdot}250}{\mathrm{DP}} - \frac{K}{\mathrm{DP}}$$

The wheel addendum, however, should be further reduced by increasing the internal diameter, to dispense with a part of the tooth flank which is not required, and the addendum of the internal gear should be made in accordance with the particulars given in Table 2.

TABLE 2.

When meshing with pinion Z_2	Values for number of teeth in Pinion Z_2						
	10	11	12	13	14	15	16 and above
Addendum "A" of wheel	·415	·445	·475	·505	·535	·570	·600

For other pitches the above values must be divided by the diametral pitch.

The internal gear should be cut with a cutter having at least 16 teeth, and the difference between the number of teeth in the gear and the number of teeth in the cutter should be at least 12 to avoid trimming when cutting, which means that using a 16 tooth cutter the minimum number of teeth in the gear would be 28.

If internal gears are required with the number of teeth less than 28 then some special arrangements will have to be made regarding the design.

In Fig. 20 is shown the teeth of an internal gear and pinion in mesh with each other 15/30 ratio, and it will be noted that the objections present in the gear pair, Fig. 17, have been removed.

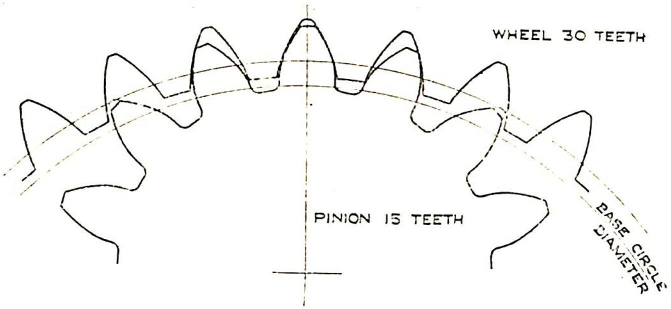


FIG. 20. SHEWING INTERNAL GEAR DESIGNED TO SPECIAL PROPORTIONS

Blank Diameters.

In view of the above remarks regarding correction the internal diameter of the internal gear and the external diameter of the pinion are calculated as follows :—

Internal Gear.

Let Z_1 = number of teeth.
DP = pitch.
A = addendum of gear tooth.
J = internal diameter of wheel.

$$\text{Then } J = \frac{Z_1}{DP} - 2A$$

External Pinion.

Let Z_2 = number of teeth.
DP = pitch.
a = pinion addendum.
j = outside diameter of pinion.

$$\text{Then } j = \frac{Z_2}{DP} + 2a$$

Backlash.

The allowable backlash is the same as was specified for external spur gears in Chapter I, Book 1, but when dealing with small pinions of less than 15 teeth it is advisable to make the chordal thickness of the pinion without any reduction, due to the fact that tips of the pinion teeth are nearly pointed. All the backlash tolerance is then allowed on the chordal thickness of the wheel tooth.

Vernier Settings.

The corrected pinions can be measured using either pitch line settings or constant chord settings but each case should be treated individually, and to calculate pitch line settings we proceed as follows—and referring to Fig. 21 let

d = pitch circle diameter of pinion.
t_p = circular thickness of pinion tooth on diameter d.
$\propto$ = half angle subtended by t_p.
a = addendum of pinion tooth.
f = correction factor.
a_c = chordal addendum of pinion tooth.
t_{pc} = chordal thickness of pinion tooth.

Then a and t_p having been previously calculated

$$\propto = \frac{t_p}{d} \text{ radians}$$

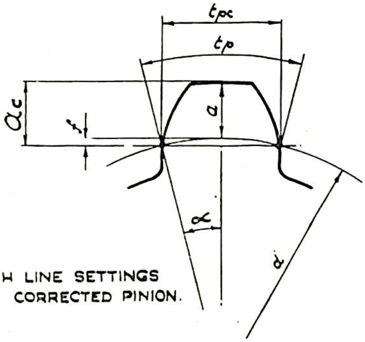


FIG 21. PITCH LINE SETTINGS FOR CORRECTED PINION.

Convert $\propto$ radians to degrees when one radian = 57·296° then
$\sin \propto d = t_{pc}$

$$\text{and } f = \frac{d}{2} \text{ versine } \propto$$

then $a_c = a + f$

From the value t_{pc} must be deducted the backlash allowance.

If constant chord settings for the pinion are desired, having previously calculated the addendum "a" and circular tooth thickness "t_p" we proceed as follows, referring to Fig. 22.

Let t_p = circular thickness of pinion tooth on diameter d.
a = addendum of pinion tooth.
ψ = pressure angle of pinion on diameter d.
a_{cc} = constant chord height setting.
t_{pcc} = constant chord thickness setting.

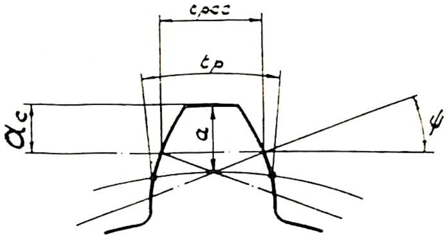


FIG. 22. CONSTANT CHORD SETTINGS FOR CORRECTED PINION.

$$\text{Then } a_{cc} = a - \frac{t_p \sin 2\psi}{4}$$

$$\text{and } t_{pcc} = t_p \cos^2 \psi.$$

Constant chord settings cannot be used for the internal gear, and it is usual to specify pitch line settings on the drawings, although in the majority of cases the gear tooth verniers cannot be used due to the internal gear being too small in diameter.

In these cases a gauge can be made as shown in Fig. 23, but it is more satisfactory to measure the gear teeth with two rollers.

To calculate pitch line settings for internal gears and referring to Fig. 24 let

D = pitch circle diameter of internal gear.
t_w = circular thickness of gear tooth on diameter D.
α = half angle subtended by t_w.
A = addendum of gear tooth.
f = correction factor.
A_c = chordal addendum of gear tooth.
t_{wc} = chordal thickness of gear tooth.

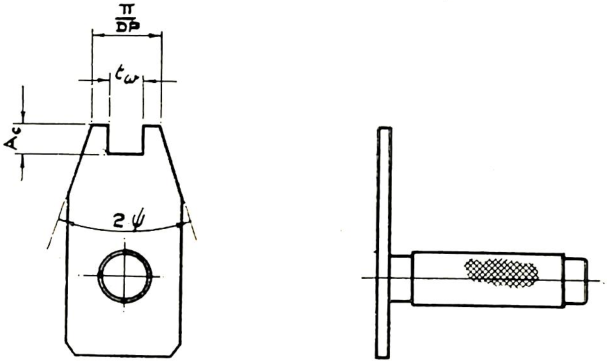


FIG. 23 GAUGE FOR PITCH LINE SETTINGS FOR INTERNAL GEAR TEETH.

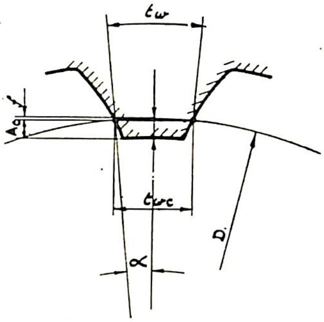


FIG. 24. PITCH LINE SETTINGS FOR CORRECTED INTERNAL GEAR.

Then $\propto$ radians $\frac{t_w}{D}$

Convert $\propto$ radians to degrees when one radian $= 57{\cdot}296°$.
then $\sin \propto D = t_{wc}$

and $f = \frac{D}{2}$ versine $\propto$

then $A_c = A - f$

Example.

An example for the detail design of an internal gear pair is given below, the particulars given being

30 teeth in wheel.
15 teeth in pinion.
16 DP pitch.
20° pressure angle.

Wheel. **Pinion.**

$D = \frac{30}{16} = 1{\cdot}875$ $d = \frac{15}{16} = 0{\cdot}9375$

then centres $C = \frac{1{\cdot}875 - 0{\cdot}9375}{2}$

$= 0{\cdot}46875 - 0{\cdot}002$

pressure angle $= 20°$ $= 20°$

$A = \frac{0{\cdot}570}{16} = 0{\cdot}0356$ $a = \frac{1{\cdot}290}{16} = 0{\cdot}0806$

$B = \frac{1{\cdot}250}{16} + \frac{0{\cdot}290}{16}$ $b = \frac{1{\cdot}250}{16} - \frac{0{\cdot}290}{16}$

$$\begin{array}{llll} B & = 0{\cdot}0781 + 0{\cdot}0181 & b & = 0{\cdot}0781 - 0{\cdot}0181 \\ & = 0{\cdot}0962 & & = 0{\cdot}060 \\ J & = \dfrac{30}{16} - 2 \times 0{\cdot}0356 & j & = \dfrac{15}{16} + 2 \times 0{\cdot}0806 \\ & = 1{\cdot}8038 & & = 1{\cdot}0987 \\ h_t & = 0{\cdot}0356 + 0{\cdot}0962 & h_t & = 0{\cdot}0806 + 0{\cdot}060 \\ & = 0{\cdot}1318 & & = 0{\cdot}1406 \end{array}$$

$$\text{Module} = \frac{1}{16} = 0{\cdot}0625 \qquad = 0{\cdot}0625$$

Note.—Addendum of both wheel and pinion does not equal the module which indicates that gears have been corrected, the pinion positive and the wheel negative.

$$t_w = \frac{3{\cdot}1416}{2 \times 16} - 2\left(\frac{0{\cdot}290}{16} \times 0{\cdot}36397\right)$$

$$t_w = 0{\cdot}0850$$

$$t_p = \frac{3{\cdot}1416}{2 \times 16} + 2\left(\frac{0{\cdot}290}{16} \times 0{\cdot}36397\right)$$

$$t_p = 0{\cdot}1113$$

Vernier Settings.

$$\begin{array}{llll} \propto \text{ radians} & = \dfrac{0{\cdot}085}{1{\cdot}875} & \propto \text{ radians} & = \dfrac{0{\cdot}1113}{0{\cdot}9375} \\ & = 0{\cdot}04533 & & = 0{\cdot}11872 \\ & = 2^\circ\ 35'\ 49'' & & = 6^\circ\ 48'\ 7'' \\ t_{wc} & = 1{\cdot}875 \times 0{\cdot}04530 & t_{pc} & = 0{\cdot}9375 \times 0{\cdot}11843 \\ & = 0{\cdot}0849 - 0{\cdot}0015 & & = 0{\cdot}1110 - 0{\cdot}0015 \\ & \qquad\quad - 0{\cdot}0030 & & \qquad\quad - 0{\cdot}0030 \\ f & = \dfrac{1{\cdot}875}{2} \times 0{\cdot}00102 & f & = \dfrac{0{\cdot}9375}{2} \times 0{\cdot}00703 \\ & = 0{\cdot}00095 & & = 0{\cdot}00329 \\ A_c & = 0{\cdot}0356 - 0{\cdot}00095 & a_c & = 0{\cdot}0806 = 0{\cdot}00329 \\ & = 0{\cdot}03465 & & = 0{\cdot}08389 \end{array}$$

Cut with cutter having 16 or 18 teeth.

CHAPTER IV.

INTERNAL GEARS.

Shaping and Slotting.

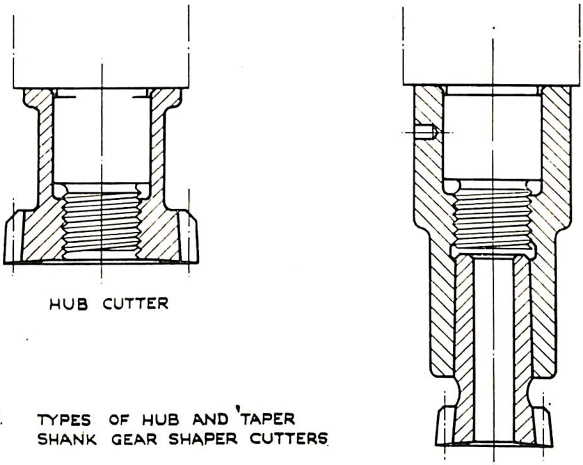
Internal gears up to approximately 24″ pitch circle diameter are usually produced on the gear shaper, while internal gears larger than this are cut on a slotting machine, the gear blank in this case being indexed by hand.

When using the gear shaper the machine is set up in a similar manner to that used for external spur gears, excepting that in the majority of cases it is necessary to use the push stroke due to the small clearances at the end of the stroke.

It is also necessary to use hub or shank cutters when it is found that a standard 3″ or 4″ pitch circle diameter is too large, based on the fact that the difference between the number of teeth in the gear to be cut, and the number of teeth in the cutter should be 12, and the minimum number of teeth in the cutter is 16, for gears of 20° pressure angle.

In Fig. 25 is shown types of hub and shank cutters generally used for cutting small internal gears, while in Fig. 26 is shown the general set up for cutting large internal gear rings, and it should be specially noted that the roll support fitted to the machine is used to support the work holding fixture during the cutting operation.

With the gear shaper method the tooth form is correctly generated, due to the fact that the tooth profiles of the cutter are



HUB CUTTER

TAPER SHANK CUTTER.

FIG. 25. TYPES OF HUB AND TAPER SHANK GEAR SHAPER CUTTERS.

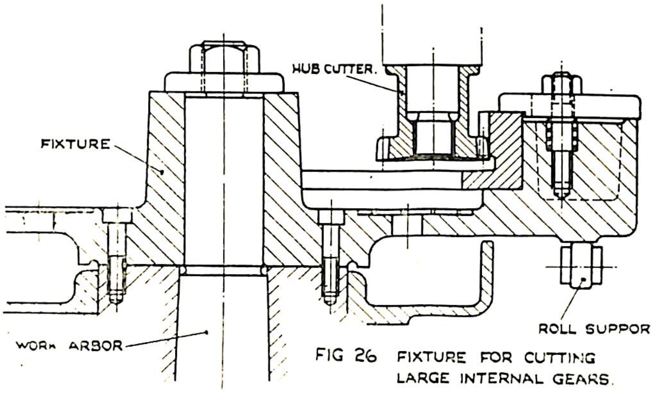


FIG 26 FIXTURE FOR CUTTING LARGE INTERNAL GEARS.

correctly generated by a grinding process after hardening, but when very large internal gears are being produced necessitating the use of a formed slotting cutter it is essential that the tooth is produced as accurately as possible.

The form of tooth then depends upon the skill of the tool draughtsman and tool maker, as the formed slotting cutter generally as shown in Fig. 27 is made by the toolmaker from a shape supplied by the tool draughtsman.

The draughtsman first makes a layout of the tooth space to be cut, on a piece of sheet zinc as shown in Fig. 28, and from the shape given the toolmaker then makes gauges, one for producing the cutter and another termed a half shape gauge for checking the tooth shape after cutting.

This gauge locates on the internal diameter of the gear as well as on one side of the tooth shape, cut by the cutter, and the gauge actually checks the relative position of the tooth space with the internal diameter, any faulty setting of the cutter or out of squareness being shown by the gauge.

In addition to supplying a layout, the tool draughtsman should also calculate the vernier settings to measure the form of the slotting cutter to ensure that the correct width and form of tooth space will be cut.

Again referring to Fig. 28 we calculate the chordal addendum and chordal thickness of the cutter when

D = pitch circle diameter of internal gear.
t_w = circular thickness of tooth.
DP = pitch of internal gear.

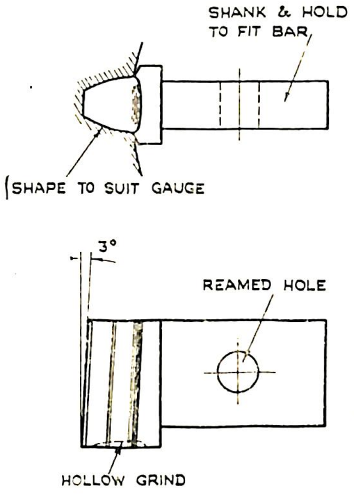


FIG. 27 SLOTTING TOOL FOR INTERNAL SPUR GEAR.

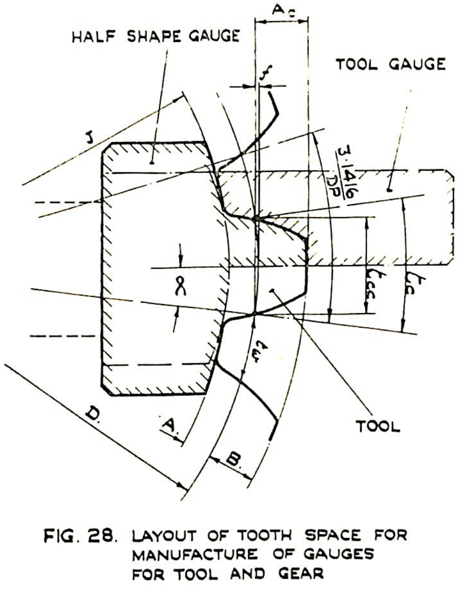


FIG. 28. LAYOUT OF TOOTH SPACE FOR MANUFACTURE OF GAUGES FOR TOOL AND GEAR

A = addendum of internal gear.
B = dedendum of internal gear.
t_c = circular thickness of cutter.
A_c = chordal addendum of cutter.
t_{cc} = chordal thickness of cutter.
f = correction factor.
α = $\frac{1}{2}$ angle subtended by t_c.

Then circular thickness of cutter tooth

$$t_c = \frac{\pi}{DP} - t_w$$

and addendum of cutter equals

B = dedendum of internal gear

and dedendum of cutter equals

A = addendum of internal gear.

Then chordal thickness of cutter tooth

$t_{cc} = \sin \alpha\, D$ + tooth thickness tolerance when

$$\alpha \text{ radians} = \frac{t_c}{D}$$

and one radian equals 57·296°.

The correction $f = \frac{D}{2}$ versine α

and chordal addendum of cutter $A_c = B + f$.

It is also usual to provide two small radii on the cutter to trim the edges of the tooth shape to blend into the internal diameter, the radius being made equal to gear tooth thickness divided by 10.

After blending in the radius the cutter shape should then be made to clear the internal diameter of the gear blank.

The cutter should be hollow ground on the front cutting face in order to produce the correct tooth space, and the top relief should be made 3° with a $1\frac{1}{2}$° side relief, but the cutter tooth section should be made the same for the full depth of cutter tooth, to enable the same tooth space to be cut for the full life of the cutter.

Example.

Taking the internal gear of 30 teeth in Chapter III, the necessary calculations for a suitable slotting tool are given below.

$$t_c = \frac{3{\cdot}1416}{16} - 0{\cdot}085$$

$$= 0{\cdot}11135$$

and addendum of cutter equals dedendum of gear B $= 0{\cdot}0962$.

$$\begin{aligned}
\propto \text{ radians} &= \frac{0{\cdot}11135}{1{\cdot}875} \\
&= 0{\cdot}05938 \\
&= 3^\circ\ 24'\ 7''
\end{aligned}$$

$$\begin{aligned}
\text{Then } t_{cc} &= 0{\cdot}05934 \times 1{\cdot}875 \\
&= 0{\cdot}11126 + 0{\cdot}003 \\
&= 0{\cdot}114 - 0{\cdot}0015 \\
f &= 0{\cdot}9375 \times 0{\cdot}00176. \\
&= 0{\cdot}00165
\end{aligned}$$

$$\begin{aligned}
\text{Then } A_c &= 0{\cdot}0962 + 0{\cdot}00165 \\
&= 0{\cdot}0978
\end{aligned}$$

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3. Deflection of Shafts and Beams (Instruction Sheet)
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5. Horse-Power of Leather Belts, etc.
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10. Shafts up to 5½-in. diameter, subjected to Twisting and Combined Bending and Twisting.
11. Shafts, 5¾ to 26 inch diameter, subjected to Twisting and Combined Bending and Twisting.
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19. Internal Expanding Brakes. Angular Proportions for Self-Balancing.
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21. Particulars for Balata Belt Drives.
22. ⅞" Square Duralumin Tubes as Struts.
23. 1" ,, ,, ,, ,,
24. ¾" Sq. Steel Tubes as Struts (30 ton yield).
25. ⅞" ,, ,, ,, (30 ton yield).
26. 1" ,, ,, ,, (30 ton yield).
27. ¾" ,, ,, ,, (40 ton yield).
28. ⅞" ,, ,, ,, (40 ton yield).
29. 1" ,, ,, ,, (40 ton yield).
30. Moments of Inertia of Built-up Sections (Tables) } Connected.
31. Moments of Inertia of Built-up Sections (Instructions and Examples)
32. Reinforced Concrete Slabs (Line Chart) } Connected.
33. Reinforced Concrete Slabs (Instructions and Examples)
34. Capacity and Speed Chart for Troughed Band Conveyors.
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105. Permissible Working Stresses in Mild Steel Struts to B.S. 449, 1948.

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